Small strain stiffness of soils

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Soil tests

- Triaxial, Simple Shear, Torsion, etc.
- Shear stress \( \tau_{13} = 0.5(\sigma_1 - \sigma_3) \)
- Shear strain \( \gamma_{13} = \varepsilon_1 - \varepsilon_3 \)
- Dynamic tests are the best means of exciting very small strains at which the response is linear-elastic.
- Hence resonant column tests; seismic velocity via bender elements.
Overview: soils at small strains

- Stiffness at very small strains $G_0$: Clayton (2010)
  - Effect of anisotropy: choosing a pertinent test
- Importance of measuring $G_0$
  - $V_s$, SASW, bender elements
- $G/G_0$ now predictable within $\pm 30\%$ (2 st. dev.)
  - Quasi-hyperbolic stress-strain curves
  - In clay depends on $w_L$: Vardanega & Bolton (2011)
  - In sand depends on $U_C$, $I_D$, $p'$: Oztoprak & Bolton (2012)
Shear stresses at small strains

- Linear elastic (maximum) shear modulus $G_0$
- $G_0$ at $\gamma < 10^{-4}\%$, $\tau < 1$ kPa
- Secant $G = \frac{\tau}{\gamma}$
- Cyclic hysteresis
- $G$ from $-\gamma$ to $+\gamma$ same as from $0$ to $+\gamma$
- Tangent stiffness $G_{\tan} = \frac{dT}{d\gamma}$ required in numerical analysis, the gradient at any point.
Elastic modulus $G_0$

- Improved approximate relationship

$$G_0 = \frac{B}{(1+e)^3} (p')^{0.5} \approx B^* (\rho_d)^3 (p')^{0.5}$$

- Parameter $B$ is not dimensionless – missing term $G_m^{0.5}$, as explained by contact mechanics.
- Values for $B$ are quoted here with $G$, $p'$ in kPa.
- $B$ of sands $\sim 50\,000$; $B$ of clays $\sim 25\,000$
  - but with factor 2 uncertainty due to characteristics of the soil grains, e.g. angularity, roughness
  - and with anisotropy, $B_{hh} \neq B_{vh}$
Database of $G_0$
Data of $G/G_0$ for sands

454 tests on dry-wet reconstituted, and undisturbed samples of clean sands, silty and gravelly sands, sandy gravels for every soil state and drainage conditions.
Data of $G/G_0$ for clays

Vardanega & Bolton (2011) IS-Seoul
Data of $G/G_0$ for sands and clays
DEM: the evolving contact network

Numerical simulation of contact force distributions: thicker lines, larger force.
(a) under isotropic stress   (b) under vertical compression
(after Thornton and Barnes, 1986)
Summary: small strain secant shear modulus $G$

- Shear stress-strain: modified hyperbola

$$\frac{G}{G_0} = \frac{1}{1+(\frac{\gamma}{\gamma_r})^a} \quad \text{for } \gamma < 0.5\%$$

- Clays: Vardanega & Bolton (2011) IS-Seoul
  - curvature parameter $a = 0.74$
  - reference strain $\gamma_r = 1.25 \ w_L \ 10^{-4}$

- Sands: Oztoprak & Bolton (2012) Geotechnique
  - curvature parameter $a = U_C^{-0.075}$
  - reference strain $\gamma_r = 8 \ e \ I_D \ 10^{-4} + U_C^{-0.3} \ p' \ 10^{-6}$
Reliability of $G/G_0$ prediction for clay

\[ G/G_0 = 1/(1+(\gamma/\gamma_{ref})^{0.74}) \]

$\gamma_{ref} = 1.25 \ W_l/1000$

Data from 66 tests on 20 clays and silts
Reliability of $G/G_0$ prediction for sand

Oztoprak & Bolton (2012) Geotechnique
Examples of small strain stiffness of sands

Rollins et al. (1998)
Saturated sand (GC=920)
Undrained RCT, p' = 200 kPa
e = 0.685, I_d = 0.40, U_c = 36

Yamashita and Toki (1995)
Undisturbed (UD) Ishikari sand
Undrained CLTXT, p' = 180 kPa
e = 0.928, I_d = 0.84, U_c = 1.7

Yamashita and Toki (1995)
Saturated Toyoura sand
Undrained CLTXT, p' = 98 kPa
e = 0.688, I_d = 0.80, U_c = 1.3

Alarcon-Guzman et al. (1989)
Dry Ottawa sand
RCT+TST, p' = 100 kPa
e = 0.635, I_d = 0.40, U_c = 1.2

Oztoprak & Bolton (2012) Geotechnique
Micromechanics of PSDs
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Constitutive relations: kinematic hardening

after Jardine (1992)
Overview: soils at small strains

- Importance of measuring $G_0$
- Effect of anisotropy: choosing a pertinent test
- DEM illustrates the origins of non-linearity
- Quasi-hyperbolic kinematic hardening function
- Predictability of $G/G_0$ using soil characterisation
- Clay and sand behave similarly